Volume: 28, Number: 2, September 2026, pp. 217-229
p-ISSN: 1410-8771, e-ISSN: 2580-4812
https://doi.org/10.37303/likhitaprajna.v28i2.993

LIKHITAPRAJNA Jurnal Ilmiah
Fakultas Keguruan dan Ilmu Pendidikan

MAPPING THE ROLE OF *CUISENAIRE RODS* ACROSS CRA STAGES IN NUMBER LEARNING FOR STUDENTS WITH INTELLECTUAL DISABILITIES

Anggy Alya Novita[1], Nia Wahyu Damayanti[2]
[1]Universitas Negeri Surabaya, Surabaya, Indonesia; anggy.23177@mhs.unesa.ac.id
[2]Universitas Negeri Surabaya, Surabaya, Indonesia; niadamayanti@unesa.ac.id

ARTICLE INFO

Article history:
Received 4 August 2026
Revised 21 August 2026
Accepted 7 September 2026

Keywords:
Cuisenaire Rods
Manipulative Media
CRA Approach
Intellectual Disabilities

To cite this article:
Novita, A., & Damayanti, N. (2026). Mapping The Role of Cuisenaire Rods Across CRA Stages in Number Learning for Students with Intellectual Disabilities. Jurnal Likhitaprajna, 28(2), 217-229. https://doi.org/10.37303/likhitaprajna.v28i2.993

ABSTRACT

Students with intellectual disabilities often experience difficulties in developing number concepts and require concrete instructional support. Cuisenaire Rods can support number learning within the Concrete–Representational–Abstract (CRA) approach, yet how their role shifts across the three stages remains insufficiently mapped. This study aims to map the role of Cuisenaire Rods across the Concrete, Representational, and Abstract stages in number learning among students with intellectual disabilities. A qualitative multiple-case study was conducted involving three students with mild intellectual disabilities at a special education school in Bojonegoro, Indonesia, selected through purposive sampling. Data were collected through observation sheets and documentation, and analyzed using data reduction, data presentation, and conclusion drawing. The findings show that Cuisenaire Rods functioned as a primary cognitive support during the Concrete stage and remained necessary during the Representational stage, albeit with varying levels of assistance across students. At the Abstract stage, their role diminished as students demonstrated greater independence: two students performed independently, while one still required assistance. These findings map a gradual shift from concrete support toward reduced dependence across CRA stages, underscoring the need for individualized instructional support in number learning for students with intellectual disabilities.

Corresponding Author:
Anggy Alya Novita
Universitas Negeri Surabaya, Surabaya, Indonesia; anggy.23177@mhs.unesa.ac.id

INTRODUCTION

Logical, analytical, and critical thinking skills in students are largely shaped through mathematics learning (Kumas, 2024). Number concepts sit at the core of this process; without a solid grasp of quantity, number relationships, and the different ways numbers can be represented, students struggle to move on to more advanced mathematical material (Flores et al., 2018; Bouck et al., 2017). In principle, mastering these basic concepts early should prepare students for more advanced mathematics later on. Students with intellectual disabilities, however, often face limitations in intellectual functioning and adaptive behavior, which makes grasping abstract concepts mathematics included particularly difficult (Bouck, Park, et al., 2017), Left unaddressed, such early difficulties can snowball, blocking access to more complex material unless teachers intervene with suitable approaches and media (Bone et al., 2021; Bouck et al., 2018).

A pre-study observation conducted on three students with mild intellectual disabilities, aged 13–14, at a special education school in Bojonegoro illustrates this gap. Each student showed a different kind of difficulty with the numbers 1–5. One could recall colors well but still struggled to write the numbers and tended to forget them quickly. Another had not yet learned to recognize or write the numbers in sequence. The third could recognize and count the numbers but needed frequent prompting to recall what had already been taught. Despite these differences, all three pointed to the same underlying need: concrete materials, repeated practice, and instruction broken into small steps so that number concepts could be genuinely understood, not just memorized (Bouck, Bassette, et al., 2017). This also confirms that helping students with intellectual disabilities reach functional mathematics competence calls for strategies tailored to each student's individual characteristics and needs (Bagadood, 2025; Martín et al., 2024; Ratnengsih et al., 2024).

The Concrete-Representational-Abstract (CRA) model is one strategy commonly used to close this gap. It moves students through three stages in sequence: handling concrete objects, working with visual representations, and finally solving problems using mathematical symbols (Bouck, Satsangi, et al., 2017). Each stage builds a bridge between real-world experience and mathematical representation, so that what students take away is conceptual understanding rather than rote procedure (Ebner et al., 2026). Numerous studies and systematic reviews have documented its effectiveness for both typically developing students and those with special needs (Agrawal & Morin, 2016; Bouck et al., 2018; Zhang et al., 2022), and several variations such as CRA/CSA and the virtual-representational-abstract sequence have also been applied successfully in inclusive classrooms (Hinton et al., 2020; Bouck et al., 2019).

Much of the CRA approach's success hinges on the manipulatives chosen for each stage. *Cuisenaire Rods* a set of colored rods whose lengths correspond to specific numerical values are a common choic (Erzuah et al., 2021). As students arrange, compare, and combine the rods, they build a hands-on sense of how numbers relate to one another before moving toward visual representations or symbols (Abreu-mendoza et al., 2021). Through activities involving arranging, comparing, and combining the rods, students gain hands-on experience in understanding relationships between numbers before transitioning to visual representations or mathematical symbols (Maulida et al., 2025), while also increasing their engagement during learning an activity that also tends to boost engagement during learning (Rufiana et al., 2022; Kanellopoulou, 2020; Khan & Khan, 2021). Beyond number introduction, *Cuisenaire Rods* have shown value in teaching arithmetic operations, fractions, and proportional reasoning across educational levels (Erzuah et al., 2021; Benson et al., 2021; Lee, 2017), and have even been adapted as a learning tool for children with Down syndrome (Vaudano et al., 2022).

Even so, most existing research has tested the CRA approach or *Cuisenaire Rods* in isolation, focusing on learning outcomes for specific topics such as fractions, multiplication, or basic addition (Flores et al., 2023). Far fewer studies look closely at how *Cuisenaire Rods* actually function at each stage of the CRA process itself, rather than simply measuring end results a gap that is even more pronounced when it comes to introducing numbers to students with intellectual disabilities in regular classrooms (Erzuah et al., 2021). In fact, understanding the patterns of media use at each stage of the CRA has the potential to provide a more detailed picture of how manipulative media help students transition from concrete experiences to abstract understanding (Zentgraf & Prediger, 2024).

The novelty of this study lies in its focus on mapping the functional role of Cuisenaire *Rods* as they shift across the three CRA stages, rather than measuring learning outcomes at a single stage or for a single topic as most prior studies have done. While earlier work has established that CRA and *Cuisenaire Rods* are effective in general (Agrawal & Morin, 2016;

Bouck, et al., 2018; Zhang et al., 2022) or effective for specific arithmetic skills (Erzuah et al., 2021; Benson et al., 2021), none has systematically traced how the level of reliance on the rods changes from serving as a primary cognitive support at the Concrete stage, to a partial scaffold at the Representational stage, to an optional aid (or none at all) at the Abstract stage within the specific context of number introduction for students with intellectual disabilities. This study further departs from the outcome-oriented, often quantitative or single-subject designs common in prior CRA research (Flores et al., 2023) by adopting a qualitative multiple-case design, which allows the varying trajectories of dependence-to-independence to be captured individually for each student rather than averaged into a single outcome measure. In doing so, this study offers a stage-by-stage, student-by-student map of the rods' role an angle that has not been explicitly documented in the existing literature on manipulative-based number learning for students with intellectual disabilities.

Based on the above description, the research question in this study is how the role of *Cuisenaire Rods* shifts across the Concrete, Representational, and Abstract stages of the CRA approach in supporting number learning among students with intellectual disabilities. This study aims to map the role of *Cuisenaire Rods* across the Concrete, Representational, and Abstract stages in number learning among students with intellectual disabilities. Consequently, this study is titled "Mapping the Role of *Cuisenaire Rods* across CRA Stages in Number Learning for Students with Intellectual Disabilities."

METHOD

Type of Research

This study uses a qualitative, multiple-case study design, which allows a phenomenon to be examined closely within its real-life context through multiple sources of evidence such as observation and documentation (Creswell, 2012). This design suits studies addressing "how" questions about a contemporary phenomenon whose boundary with its context is not clearly evident (Yin, 2018) as is the case with how *Cuisenaire Rods* function as students move through the CRA stages.

The three subjects were treated as individual cases, first analyzed separately and then compared through cross-case analysis to identify patterns in the use of *Cuisenaire Rods* across the Concrete, Representational, and Abstract stages. This follows the replication logic of multiple-case designs, in which cases are selected to reveal either similar patterns (literal replication) or contrasting but theoretically explainable patterns (theoretical replication), rather than to achieve statistical generalization (Yin, 2018). Consistent tendencies across the three subjects thus lend stronger support to the study's interpretations despite the small number of cases.

Three students with mild intellectual disabilities at a special education school in Bojonegoro were selected through purposive sampling, based on documented difficulty recognizing numbers 1–5, the ability to follow simple instructions, and willingness to complete the full learning sequence (Ting et al., 2025; Susanto et al., 2025). This selection reflects case study logic, in which cases are chosen for their theoretical relevance to the research focus rather than statistical representativeness (Yin, 2018).

Research Subjects

Research subjects were selected using purposive sampling, based on the alignment of their characteristics with the research focus rather than statistical representativeness (Yin, 2018). The criteria included mild intellectual disability, documented difficulty recognizing the numbers 1-5, the ability to follow simple instructions, and willingness to complete the entire learning process. Three subjects meeting these criteria were selected at the SLB in Bojonegoro: NHP (14 years old, 9th grade), KNS (14 years old, 8th grade), and APW (13 years old, 8th grade). All three were in Curriculum Phase A, with learning achievement comparable to early elementary grades, and had been diagnosed with mild intellectual disability affecting memory and verbal articulation, though all could understand simple instructions and respond verbally. Each had previously received basic counting instruction (1-5) but still struggled to recognize and write number symbols and to match quantities with their corresponding symbols.

Three subjects were included in line with the replication logic of multiple-case designs, in which additional cases serve to strengthen confidence in findings through pattern replication rather than to increase sample size (Yin, 2018); this number also allowed each case to be examined in sufficient depth given the study's time and resource constraints. Subjects with mild intellectual disability were specifically targeted because their capacity to follow instructions and respond verbally, combined with their documented difficulty grasping abstract number concepts, made them well suited to revealing in detail how *Cuisenaire Rods* function across the CRA stages.

Ethical Considerations

As a matter of ethical consideration, this study obtained permission from the school where the research was conducted. Prior to the study, the researcher also obtained written consent from the parents/guardians of each participant, as well as assent from each participant. The identities of the three participants were disguised using initials, and the visual documentation used was edited to maintain confidentiality, with publication permission obtained from parents/guardians and the school.

Research Instruments

The instruments used in this study consisted of *Cuisenaire Rods* as the primary object of analysis, a task-analysis-based observation sheet as the main data collection tool, and field notes together with visual documentation as supporting instruments. *Cuisenaire Rods* of uniform size and color were used across all three CRA stages: Concrete (observing, counting, and selecting rods according to instructions), Representational (redrawing the number of rods that had been manipulated), and Abstract (writing number symbols based on a drawing of rods without concrete media). Uniform size and color were deliberately chosen so that students could focus solely on counting and matching quantities with number symbols, rather than being distracted by variation in size or color a consideration particularly relevant for students with intellectual disabilities, who tend to have limited attentional capacity. The rods' concrete, three-dimensional form also allowed students to hold, move, and count them directly, providing the hands-on experience needed before progressing toward visual and symbolic representation.

The main instrument for data collection was a task-analysis-based observation sheet used to record patterns of *Cuisenaire Rods* use as well as the form and level of assistance provided at each CRA stage, adapted from (Aspasyah et al., 2026). The sheet employed three assessment categories: Independent (M), assigned when a student completed a task without any verbal or physical assistance; With Assistance (B), assigned when a student required verbal assistance (e.g., repeated instructions or guiding questions) and/or minor physical assistance; and Not Performed (TD), assigned when a student was unable to complete the task even with assistance, following the same categorization (Aspasyah et al., 2026). This instrument was chosen because it allowed the shifting level of support each student needed to be recorded systematically and consistently across sessions and stages data that were essential for capturing how the role of *Cuisenaire Rods* changed as students moved from the Concrete to the Abstract stage.

An additional instrument was used to measure each student's ability to recognize the numbers 1-5 after completing the CRA learning sequence with *Cuisenaire Rods*. This instrument was developed based on the three CRA stages and adapted from Mathematics for Elementary Teachers: A Contemporary Approach and from Flores, Hinton, and Taylor's (2018) study on CRA-based fraction intervention using concrete manipulatives, then further adjusted to fit the use of *Cuisenaire Rods* in number learning for students with intellectual disabilities.

This instrument was chosen because it provided a structured way to corroborate the observational findings with a direct measure of each student's number-recognition outcome at the end of the learning sequence.

Field notes and visual documentation (photographs and video recordings) served as supporting instruments. These captured contextual details not fully reflected in the observation sheet and provided material for resolving discrepancies between observers as well as for triangulating the observational data.

Data Collection Techniques

The study was conducted from July 13 to 17, 2026, at Tumbrasanom State Special Education School (SLB Negeri Tumbrasanom) in Kedungadem, Bojonegoro. Each subject took part in three CRA sessions using *Cuisenaire Rods*, with each session lasting 35 minutes. Two observers the subject's classroom teacher and a mathematics teacher carried out direct observation after being briefed beforehand on the learning procedures and the observation sheet, ensuring that both assessed patterns of media use consistently across stages. Whenever their records differed, the researcher and the observers discussed the discrepancy and reviewed the video footage together until reaching agreement. Photographs, video recordings, and field notes supplemented the observation sheets. Triangulating the observation results with the field notes and video documentation helped confirm that the data on media use were accurate and consistent before proceeding to analysis.

Data Analysis Techniques

The data were analyzed following the three stages of qualitative data analysis: data reduction, data presentation, and drawing conclusions (Miles et al., 2014). In the reduction stage, findings from the observation sheets, field notes, and video documentation were filtered down to what mattered most how the *Cuisenaire Rods* were actually used and how the three subjects responded at each CRA stage. This reduced data was then written up as descriptive narratives of media-use patterns at each stage, before being interpreted against relevant theory to draw conclusions about the rods' contribution to students' understanding of numbers.

RESULTS AND DISCUSSION

RESULTS

This section reports how *Cuisenaire* Rods were used at each stage of the CRA process in introducing numbers to students with intellectual disabilities. Rather than focusing solely on the general implementation of the learning process, the data are organized around patterns of media use at each stage, so as to illustrate how the rods were utilized, relied upon, and responded to by the three subjects as they progressed from concrete manipulation toward abstract symbolic understanding. This framing makes it possible to trace how each subject's reliance on the media shifted over time, along with the type and intensity of assistance the researcher provided at each stage details that say as much about a student's readiness to advance as the outcome itself. These patterns set up the stage-by-stage discussion that follows, comparing the three subjects' trajectories against the broader aim of the CRA approach: moving students from concrete experience toward abstract, self-sustained understanding of numbers.

Table 1. Patterns of *Cuisenaire Rods* Media Use and Students' Level of Independence

Subject	Media Use in the Concrete Stage	Media Use in the Representational Stage	Media Use in the Abstract Stage
Subject 01 (NHP)	B	B	M
Subject 02 (KNS)	B	B	B
Subject 03 (APW)	B	B	M

Notes:
M = Independent

B = With Assistance
TD = Not Performed

Use of *Cuisenaire* Rods in the Concrete Stage

Figure 1. Use of *Cuisenaire Rods* Media in the Concrete Stage

a.) Subject 01 (NHP)

At this stage, *Cuisenaire Rods* were used as objects to be observed, counted, and picked up according to the researcher's instructions. Subject 01 showed attention to the rods and was able to follow instructions step by step, but still required verbal assistance in the form of repeated instructions as well as occasional light physical assistance to guide the position of the rods to make them easier to count. Subject 01's use of the rods at this stage falls into the "With Assistance" (B) category.

b.) Subject 02 (KNS)

The material was used in the same way that is, observed, counted, and picked up according to instructions. Subject 02 showed attention to the material but experienced more fundamental difficulties in counting sequentially and determining how many rods to pick up, thus requiring verbal assistance in the form of repeated instructions as well as more intensive physical assistance compared to other subjects. Subject 02's use of the material at this stage falls into the "With Assistance" (B) category.

c.) Subject 03 (APW)

Subject 03's pattern of media use was similar to that of Subject 01: able to observe and follow instructions step by step, but still requiring verbal assistance in the form of repeated instructions and guiding questions, as well as occasional light physical assistance when picking up rods. Subject 03's use of the media at this stage falls into the "With Assistance" (B) category.

Use of *Cuisenaire Rods* at the Representational Stage

Figure 2. Use of *Cuisenaire Rods* as a Teaching Aid in the Representational Stage

a.) Subject 01 (NHP)

Subject 01 still required previously manipulated *Cuisenaire Rods* as the primary reference for representing the number of objects. However, the subject was able to understand the instructions and complete the drawing based on their own observations without requiring additional physical assistance from the researcher; the resulting drawing corresponded to the number of rods previously used. The use of media at this stage falls under the "With Assistance" (B) category.

b.) Subject 02 (KNS)

Subject 02 also still requires *Cuisenaire Rods* as a concrete medium, and their dependence on them is higher than that of other subjects; Subject 02 requires assistance to recall the number of rods previously counted, as well as light physical assistance when experiencing difficulties during the drawing process. The use of materials at this stage falls under the "With Assistance" (B) category.

c.) Subject 03 (APW)

Similar to Subject 01 and Subject 02, Subject 03 still needs *Cuisenaire Rods* as a reference, but is able to reproduce the number of rods independently without physical assistance, and the resulting drawing matches the number of rods from the previous stage. The use of teaching aids at this stage falls under the “With Assistance” (B) category.

Use of *Cuisenaire Rods* at the Abstract Stage

Figure 3. Use of *Cuisenaire Rods* as a Teaching Aid in the Abstract Stage

a.) Subject 01 (NHP)

Physical media are no longer used directly; instead, the subject is presented with images of rods on the assessment sheet and asked to write the corresponding number symbols. Subject 01 is able to connect the pictorial representations with the number symbols independently, without verbal or physical assistance. The use of media-based experiences at this stage falls under the Independent (M) category.

b.) Subject 02 (KNS)

Subject 02 was asked to write the number symbols based on the rods images presented but still had difficulty recognizing some number symbols, causing them to pause several times before answering. The researcher provided verbal assistance in the form of guiding questions as well as light physical assistance to help the subject complete the task. The use of media-based learning outcomes at this stage falls under the "With Assistance" (B) category.

c.) Subject 03 (APW)

Similar to Subject 01, Subject 03 was presented with images of the rods and asked to write the corresponding number symbols. The subject observed the images attentively before writing the answer, and was able to connect the visual representation with the correct number symbol independently, without requiring verbal or physical assistance from the researcher. This result suggests that the accumulated experience of using the media in the previous stages had been successfully internalized into abstract symbolic understanding. The use of prior media experience at this stage falls under the Independent (M) category.

DISCUSSION

The following discussion analyzes the patterns of *Cuisenaire Rods* media use at each CRA stage based on the data in Table 1, which shows that students' dependence on the media does not decrease linearly, but rather persists until the Representational stage before finally diminishing at the Abstract stage and even then, only for some subjects. This pattern is discussed further below in relation to the concept of representational transition, which frames each CRA stage not merely as a sequence of materials but as a translation process between concrete, visual, and symbolic forms of representing number (Dyrvold, 2018; Andriani et al., 2022).

Media Use in the Concrete Stage: Initial Dependence on Concrete Objects

At this stage, all three subjects NHP, KNS, and APW were in the "With Assistance" (B) category when using *Cuisenaire Rods* to observe, count, and pick up objects according to instructions. This consistent pattern indicates that the presence of concrete media does not automatically enable students with intellectual disabilities to use it independently from the start; the media still needs to be accompanied by repeated verbal instructions and light physical assistance so that students can translate instructions into actions of counting and picking up objects (Root et al., 2021). This finding is consistent with the characterization of intellectual disability as a limitation not only in abstract reasoning but also in sustaining attention and following multi-step instructions, both of which are required even for concrete manipulative tasks (AAIDD, 2021). In other words, the difficulty observed here is not solely a function of the abstractness of number concepts, but also of the executive and attentional demands embedded in the concrete task itself a distinction that is often underemphasized in studies that treat the Concrete stage as inherently "easy" for students with intellectual disabilities (S. & Sengar, 2026).

This indicates that challenges faced by students with intellectual disabilities do not only arise at the symbolic-abstract stage but also at even the most concrete stages; thus, consistent teacher guidance remains a prerequisite for the successful use of these materials, in line with the need for tailored mathematics learning strategies for students with intellectual disabilities (Hakim & Mulyatna, 2023; Barr & Mavropoulou, 2019). Viewed through the lens of representational transition, the assistance required at this stage can be understood as scaffolding the very first translation students must perform: from a verbal instruction to a physical, concrete action (Andriani et al., 2022). Because this initial translation step is itself effortful for students with intellectual disabilities, it lends support to the argument that individualized, gradual accommodation should begin from the earliest stage of the CRA sequence, rather than being reserved for the later, more symbolic stages (Bagadood, 2025).

Use of Media in the Representational Stage: Media Have Not Yet Moved Beyond Their Concrete Role

Contrary to the common assumption that the representational stage marks students' detachment from physical media, the data in this study show that all three subjects still fall into the "With Assistance" (B) category. The previously manipulated *Cuisenaire Rods* remain the primary reference when representing the quantity of objects, although the form and intensity of the assistance vary. Subjects 01 and 03 required relatively minimal assistance (verbal, without physical assistance), while Subject 02 required verbal assistance to recall the number of rods as well as physical assistance while drawing. This pattern indicates that the transition from concrete experience to visual representation is not an automatic or instantaneous process, but rather a gradual one in which concrete media continue to function as a cognitive "anchor" for students with intellectual disabilities before visual representation can stand on its own (Park et al., 2020; Root et al., 2021).

These findings differ slightly from the general pattern observed in previous research, which positions the Representational stage as the point at which some students are already capable of independence (Flores & Hinton, 2022), and instead confirm that, in the case of number recognition for students with intellectual disabilities, reliance on concrete objects may persist longer than anticipated. This extended reliance also aligns with evidence that representational competence typically requires explicit and gradual training rather than developing spontaneously once concrete understanding is in place a pattern documented even among pre-service teachers, who might be assumed to have stronger abstract reasoning skills than the students in this study (Pehlivan & Aslan-tutak, 2019). Taken together, this suggests that the boundary between the Concrete and Representational stages, as commonly depicted in CRA literature, may be more porous for students with intellectual disabilities than the standard three-stage model implies, and that instructional pacing between these two stages may need to be slower and more individualized than typically prescribed (Anwar & Trends, 2025).

Media Use in the Abstract Stage: The Turning Point for Independence Among Subjects

At this stage, physical media are completely replaced by images as symbolic representations, and this is where patterns of media use begin to diverge among subjects. Subject 01 and Subject 03 were able to independently connect images of rods with number symbols, indicating that the accumulated experience of using media in the two previous stages albeit with assistance was ultimately successfully internalized into a self-sustaining symbolic understanding, in line with the ultimate goal of the CRA approach (Bouck, Park, et al., 2018). This reflects the successful completion of a translation chain from object to image to symbol, in which competence at the final link depends cumulatively on practice at earlier links rather than on abstract-reasoning ability alone (Nielsen & Bostic, 2018).

Conversely, Subject 02 still required assistance at this stage, indicating that for students with more fundamental number recognition difficulties, experience with the media was not yet sufficient to facilitate a transition to abstract symbols without additional guidance; thus, more intensive repetition and support remained necessary (Root et al., 2021), as also found in the application of the virtual-representational-abstract sequence (Bouck et al., 2019), as well as the CRA/CSA approach in inclusive classrooms (Hinton et al., 2020).

The Contribution of Multisensory Media and Individual Variations in Its Use

Overall, the use of *Cuisenaire Rods* throughout the CRA stages provides a multisensory learning experience visual, tactile, and kinesthetic that helps all three subjects build an understanding of abstract number concepts through concrete experiences (Abreu-mendoza et al., 2021; Erzuah et al., 2021). This multisensory quality is consistent with evidence that manipulative-based instruction not only supports conceptual understanding but also sustains attention and engagement among students with intellectual disabilities, who often show reduced engagement under purely verbal or symbolic instruction (Khan & Khan, 2021; Kanellopoulou, 2020). The rods' color-coding and tactile handling in particular appear to have supported students' engagement independently of their level of independence in completing each task, suggesting that motivational and cognitive support functions of the media may operate somewhat separately from one another (Vaudano et al., 2022).

However, the patterns in Table 1 indicate that the media's contribution to students' independence is cumulative and gradual, not instantaneous: dependence on the media persists through the representational stage for all subjects and only diminishes during the abstract stage and even then, only for subjects with relatively higher initial readiness. This variation underscores that the effectiveness of concrete manipulative media cannot be separated from students' individual characteristics and prior abilities (Firma & Astuti, 2025), thus, the amount and form of support accompanying media use must be designed in a gradual and adaptive manner, rather than assumed to decrease uniformly as students progress through the CRA stages.

CONCLUSION

This study achieved its objective of mapping the role of *Cuisenaire Rods* at each stage of the Concrete-Representational-Abstract (CRA) approach in introducing numbers to students with intellectual disabilities. The findings indicate that students' reliance on the media does not decrease linearly as they progress through the CRA stages but persists until the Representational stage, and only diminishes at the Abstract stage, where two subjects were able to work independently, while one subject with more fundamental difficulties in number recognition continued to require support.

Thus, it can be concluded that *Cuisenaire Rods* serve as a cognitive anchor that helps students with intellectual disabilities build an understanding of numbers gradually, and their contribution to independent learning is cumulative and depends on the characteristics and initial abilities of each student. The effectiveness of using concrete manipulatives in the CRA approach should not be assessed solely based on final learning outcomes but also on patterns of use at each stage.

As a follow-up, teachers are advised not to rush to remove concrete manipulatives during the representational stage, given that most students still need them as a reference. Future research is recommended to examine patterns of using similar manipulatives across a broader scope of number concepts or with other types of manipulatives, as well as to involve a larger number of participants.

ACKNOWLEDGMENTS

I would like to thank the Special Education School (SLB) in Bojonegoro for granting permission to carry out this study, and the teachers and students who took part and cooperated throughout the learning activities. Thanks are also due to the Mathematics Education Study Program at Universitas Negeri Surabaya for its support in completing this research.

REFERENCES

Abreu-Mendoza, R. A., Coulanges, L., Ali, K., Powell, A. B., & Rosenberg-Lee, M. (2021). From non-symbolic to symbolic proportions and back: A Cuisenaire Rod proportional reasoning intervention enhances continuous proportional reasoning skills. *Frontiers in Psychology, 12*, 633077. https://doi.org/10.3389/fpsyg.2021.633077

Agrawal, J., & Morin, L. L. (2016). Evidence-based practices: Applications of concrete representational abstract framework across math concepts for students with mathematics disabilities. *Learning Disabilities Research & Practice, 31*(1), 34–44. https://doi.org/10.1111/ldrp.12093

Andriani, P., Kurniawati, K. R. A., & Afriyani, D. (2022). A framework for assessing translation among multiple representations. *JTAM (Jurnal Teori dan Aplikasi Matematika), 6*(2), 321–330. https://doi.org/10.31764/jtam.v6i2.7193

American Association on Intellectual and Developmental Disabilities. (2021). *Intellectual disability: Definition, diagnosis, classification, and systems of supports* (12th ed.). American Association on Intellectual and Developmental Disabilities.

Anwar, R., Rahmawati, D., & Wulandari, A. (2025). Trends in research on mathematical representation in mathematics learning: A systematic literature review. *International Journal of Education in Mathematics, Science and Technology, 13*(3), 698–714. https://doi.org/10.46328/ijemst.4848

Aspasyah, J., Arianto, H., & Damayanti, N. W. (2026). Early numeracy learning based on Vignette Activity Sequence (VAS) using number boards for students with cerebral palsy. *Likhitaprajna Jurnal Ilmiah Fakultas Keguruan dan Ilmu Pendidikan, 28*(1), 111–124.

https://doi.org/10.37303/likhitaprajna.v28i1.973
Bagadood, N. H. (2025). Tailoring instructional strategies to improve functional mathematics competence in children with intellectual and developmental disabilities. *American Journal of Psychiatric Rehabilitation, 28*(1). https://doi.org/10.69980/ajpr.v28i1.111
Barr, F., & Mavropoulou, S. (2019). Curriculum accommodations in mathematics instruction for adolescents with mild intellectual disability educated in inclusive classrooms. *International Journal of Disability, Development and Education*. https://doi.org/10.1080/1034912x.2019.1684457
Benson, I., Marriott, N., & McCandliss, B. (2021). *Engaging algebra early through manipulatives: Reappraising Cuisinaire-Gattegno rods*. https://doi.org/10.31234/OSF.IO/ZASB9
Bone, E. K., Bouck, E. C., & Satsangi, R. (2021). Comparing concrete and virtual manipulatives to teach algebra to middle school students with disabilities. *Exceptionality, 30*(1), 1–17. https://doi.org/10.1080/09362835.2021.1938057
Bouck, E. C., Bassette, L., Shurr, J., Park, J., Kerr, J., & Whorley, A. (2017). Teaching equivalent fractions to secondary students with disabilities via the virtual–representational–abstract instructional sequence. *Remedial and Special Education, 32*(4), 220–231. https://doi.org/10.1177/0162643417727291
Bouck, E. C., Park, J., & Nickell, B. (2017). Using the concrete-representational-abstract approach to support students with intellectual disability to solve change-making problems. *Research in Developmental Disabilities, 60*, 24–36. https://doi.org/10.1016/j.ridd.2016.11.006
Bouck, E. C., Park, J., Shurr, J., Bassette, L., & Whorley, A. (2018). Using the virtual-representational-abstract approach to support students with intellectual disability in mathematics. *Focus on Autism and Other Developmental Disabilities, 33*(4), 237–248. https://doi.org/10.1177/1088357618755696
Bouck, E. C., Park, J., & Shurr, J. (2019). Using the virtual-representational instructional sequence to support the acquisition and maintenance of mathematics for students with intellectual disability. *International Journal of Developmental Disabilities, 67*(3), 217–228. https://doi.org/10.1080/20473869.2019.1640999
Bouck, E. C., Satsangi, R., & Park, J. (2017). The concrete–representational–abstract approach for students with learning disabilities: An evidence-based practice synthesis. *Remedial and Special Education, 39*(4), 211–228. https://doi.org/10.1177/0741932517721712
Creswell, J. W. (2012). *Qualitative inquiry and research design: Choosing among five approaches* (3rd ed.). SAGE Publications.
Dyrvold, A. (2018). *Conceptualising translations between representations*. https://www.semanticscholar.org/paper/ce605c30a106d8bde6c1f0129fa1f75a04e53905
Ebner, S., Romines, G., & Kathleen, P. (2026). Combining the concrete representational abstract approach and video modeling to increase basic addition skills for first grade students. *Journal of Behavioral Education, 35*(2), 1024–1046. https://doi.org/10.1007/s10864-026-09623-0
Erzuah, E., Kwakye, D. O., Issah, J. A., Ayipala, J. A., Redeemer, S., Susuoroka, G., & Mohammed, A. A. (2021). *Perception of Teacher Trainees on the use of Cuisenaire Rod in Learning Fractions*. *8*(5), 15–24.
Fima, D., & Astuti, E. Y. (2025). *Enhancing Basic Arithmetic Skills in Children with Mild Intellectual Disabilities Through Puzzle Media*. *10*(5), 802–810.https://doi.org/10.31004/jele.v10i5.1340
Flores, M. M., & Hinton, V. M. (2022a). The effects of a CRA-I intervention on students' number sense and understanding of addition. https://doi.org/10.1177/07419325211038009
Flores, M. M., Hinton, V. M., & Schweck, K. B. (2023). Using CRA-I to teach fraction and

decimal concepts to students with learning disabilities. *Learning Disability Quarterly*. https://doi.org/10.1177/07319487231176545
Flores, M. M., Hinton, V. M., & Taylor, J. J. (2018). CRA fraction intervention for fifth-grade students receiving tier two interventions. *Preventing School Failure: Alternative Education for Children and Youth, 62*(3), 198–213. https://doi.org/10.1080/1045988X.2017.1414027
Hakim, A. R., & Mulyatna, F. (2023). *Sejarah Matematika : Perkembangan Bilangan Matematika Empiris*. *80*, 471–478.
Hinton, V. M., Gibbs, A., & Franklin, T. M. (2020). The effects of CRA/CSA explicit instruction for students with and without disabilities taught in an inclusive setting. *Journal of the American Academy of Special Education Professionals (JAASEP)*. https://doi.org/10.64546/jaasep.414
Kanellopoulou, E. (2020). Learning counting skills through CRA: The case of children with intellectual disability. *Open Access Library Journal*. https://doi.org/10.4236/oalib.1106152
Khan, R., & Khan, M. (2021). Concrete-representational-abstract and multisensory strategies: An inclusive approach to mathematics. *Asia Pacific Journal of Developmental Differences*. https://doi.org/10.3850/s2345734121000139
Kumas, O. A. (2024). The power of digital story in early mathematics education: Innovative approaches for children with intellectual disabilities. *PLOS ONE*, 1–17. https://doi.org/10.1371/journal.pone.0302128
Lee, J. (2017). A critical review on the use of Cuisenaire rods in learning of fraction. *The Mathematics Educator, 56*(2), 193–212. https://doi.org/10.7468/mathedu.2017.56.2.193
Martín, C. S., Martinez, M. V., Rojas, F., Ramírez, C., & Cáceres, A. (2024). Mathematics education for students with intellectual disabilities: Beliefs of Chilean special education teachers. *International Journal of Disability, Development and Education*. https://doi.org/10.1080/1034912X.2024.2427614
Maulida, T., Budi, S., Efrina, E., & Mahdi, A. (2025). Meningkatkan kemampuan mengenal lambang bilangan melalui Cuisenaire Rods pada disabilitas intelektual ringan. *Pedagogik Journal of Islamic Elementary School, 8*(3), 1266–1278. https://doi.org/10.24256/pijies.v8i3.9107
Miles, M. B., Huberman, A. M., & Saldaña, J. (2014). *Qualitative data analysis: A methods sourcebook* (3rd ed.). SAGE Publications.
Musser, G. L., Peterson, B. E., & Burger, W. F. (2013). *Mathematics for elementary teachers: A contemporary approach* (10th ed.). John Wiley & Sons.
Nielsen, M., & Bostic, J. (2018). Connecting and using multiple representations. *Mathematics Teaching in the Middle School, 23*(7), 386–393. https://doi.org/10.5951/mathteacmiddscho.23.7.0386
Park, J., Bouck, E. C., & Fisher, M. H. (2020). Using the virtual–representational–abstract with overlearning instructional sequence to students with disabilities in mathematics. https://doi.org/10.1177/0022466920912527
Pehlivan, Z., & Aslan-Tutak, F. (2019). *A look into Turkish preservice teachers' translation skills: Case for model representations*. https://www.semanticscholar.org/paper/9246b5a2098418c4bca87eeb7a4f34cd5050f7aa
Ratnengsih, E., Nurihsan, J., Rochyadi, E., Sunardi, & Ediyanto. (2024). Mathematics education for students with intellectual disabilities: A study on thematic books utilization in special schools. *International Journal of Education and Practice, 12*(3). https://doi.org/10.18488/61.v12i3.3816
Root, J. R., Cox, S. K., Gilley, D., & Wade, T. (2021). Using a virtual-representational-abstract integrated framework to teach multiplicative problem solving to middle school students with developmental disabilities. *Journal of Autism and Developmental Disorders, 51*(7),

2284–2296. https://doi.org/10.1007/s10803-020-04674-2
Rufiana, I. S., Ekayanti, A., Munawaroh, I. T., Putri, F. I. P., Yuwiarti, I., & Wulantari, V. (2022). Pembelajaran bilangan bulat dengan menggunakan Cuisenaire Rods. *Jurnal Silogisme: Kajian Ilmu Matematika dan Pembelajarannya, 7*(2), 127–137. https://doi.org/10.24269/silogisme.v7i2.6457
S., B., & Sengar, K. S. (2026). A systematic review of effectiveness of Concrete Representational Abstract (CRA) intervention for individuals with mathematical disability. *International Journal of Drug Delivery Technology*. https://doi.org/10.25258/ijddt.16.50s.97
Susanto, M. R., Nasution, U. B., & Supriyono, L. A. (2025). *Metodologi penelitian lanjutan*. PT. Green Pustaka Indonesia.
Ting, H., Memon, M. A., Thurasamy, R., Cheah, J., Management, T., Lumpur, K., & Kingdom, U. (2025). *Snowball Sampling : A Review and Guidelines for Survey*. *15*(1), 1–15.https://doi.org/10.14707/ajbr.250186
Vaudano, G., Casal, J., & Picado, L. (2022). The mathematical development of children with Down syndrome: The adapted Cuisenaire material as a learning facilitator. *International Journal of Social Science Research and Review, 5*(10). https://doi.org/10.47814/ijssrr.v5i10.649
Yin, R. K. (2018). *Case study research and applications: Design and methods* (6th ed.). SAGE Publications.
Zentgraf, K., & Prediger, S. (2024). Demands and scaffolds for explaining the connection of multiple representations: Revisiting the bottle-filling task. *Journal of Mathematical Behavior, 73*, 101118. https://doi.org/10.1016/j.jmathb.2023.101118
Zhang, S., Yu, S., Xiao, J., Liu, Y., & Jiang, T. (2022). The Effects of Concrete - Representational - Abstract Sequence Instruction on Fractions for Chinese Elementary Students with Mathematics Learning Disabilities. *International Journal of Science and Mathematics Education*, *20*(7), 1481–1498. https://doi.org/10.1007/s10763-021-10215-9